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On tests of local realism by CP-violation parameters of K^0 mesons

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Abstract. Recently various papers have proposed to test local realism (LR) by considering electroweak CP-violation parameters values in neutral pseudoscalar meson systems. Considering the large interest for a conclusive test of LR and the experimental accessibility to these tests, in this paper we critically consider these results showing how they, albeit that they are very interesting, require anyway additional assumptions and therefore cannot be considered conclusive tests of LR.

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1 Introduction

The problem if a local realistic theory can reproduce the standard quantum mechanics (SQM) results originates from the early days of this theory. Even if now SQM is confirmed by a huge amount of data and represents one of the pillars of modern physics, the fundamental quest to a possible realistic theory reproducing its results remains open (for a very recent review see [1]. A general discussion of this problems can also be found, for example, in the textbooks of [2,3]).

The earliest discussion of this problem dates from 1935, when Einstein–Podolsky–Rosen [4] suggested that quantum mechanics could be an incomplete theory, representing a statistical approximation of a complete deterministic theory in which observable values are fixed by some hidden variable.

In 1964 Bell [5] discovered that any realistic local hidden variable theory (LHVT) must satisfy certain inequalities which can be violated in SQM leading to the possibility of an experimental test of the validity of SQM as compared to LHVT.

In little more detail, Bell analyzed the correlation properties of a two particles entangled system, where an entangled state is defined as a state of two or more particles which cannot be factorised in single-particle states, namely (as stated in the original Schrödinger definition [6]) a compound state whose subsystems are not probabilistically independent[1]. The result of this analysis was that one had shown how quantum correlation functions can violate some inequality that is always satisfied by *every* local realistic theory.

We would like to emphasize that the great beauty of this theorem resides in demonstrating in complete generality that every local realistic theory cannot reproduce all the results of SQM and therefore one can exclude a whole class of theories with a single experiment: no other hypothesis beyond locality (namely the request of no superluminal "connection" between subsystems) is introduced[2]. Considering the extreme generality of this theoretical result, one must therefore be careful not to introduce additional hypotheses when implementing an experiment for testing it.

Since then, many experiments (mainly based on entangled photon pairs) have been addressed to test the Bell inequalities [8–10], leading to a substantial agreement with standard quantum mechanics and strongly disfavoring LHVT. The request of having well space-like separated measurements (necessary for excluding every direct influence of measurements results[3], the so called *locality loophole*) has been well verified [8,10]. However, so far, no experiment has yet been able to exclude definitively such theories, since one has always been forced to introduce a further additional hypothesis [11,13], due to the low total detection efficiency, stating that the observed sample of particle pairs is a faithful subsample of the whole (*detec-*

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[1] For instance, a singlet of two spin 1/2 particles.

[2] On the other hand, the Bell theorem does not concern non-local hidden variables theories. For a recent progress toward an experimental verification of one of them see [7] and therein.

[3] In discussing a completely general *local* hidden variable theory one envisages whatever influence between subsystems with the only condition that it is not superluminal [5,1]. When discussing a completely general *non-local* hidden variable theory this condition is further relaxed to the request that an eventual superluminal influence among subsystems does not allow for faster than light information transmission [1].

tion or efficiency loophole)[4]. It has been shown that this loophole could be eliminated only by reaching a detection efficiency of 0.8284 when using maximally entangled states or 0.67 for non-maximally entangled ones [12]. For the sake of clarity, it must be emphasized that this result is completely general and does not depend on the specific physical system nor on the detection scheme.

Due to this limitation of all the experiments performed up to now, the quest for new experimental configurations able to overcome the detection loophole is of the utmost interest.

Recently, many papers [20] have been devoted to a study of the possibility of realizing a conclusive test by the use of pseudoscalar meson pairs as $K^0\bar{K^0}$ or $B^0\bar{B}^0$. If the pair is produced by the decay of a particle at rest in the laboratory frame (as the ϕ at Daϕne), the two particles can easily be separated to a relatively large distance allowing for a space-like separation of the two subsystems and permitting an easy elimination of the locality loophole, i.e. realizing two completely space-like separated measurements on the two subsystems (where the space-like separation must include the setting of the experimental apparatuses too). A very low noise is expected as well.

The idea is to use entangled states (i.e. non-factorizable in single-particle states) of the form

$$|\Psi\rangle = \frac{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle}{\sqrt{2}} = \frac{|K_\mathrm{L}\rangle|K_\mathrm{S}\rangle - |K_\mathrm{S}\rangle|K_\mathrm{L}\rangle}{\sqrt{2}}, \quad (1)$$

where $|K^0\rangle$ and $|\bar{K}^0\rangle$ are the particle and antiparticle related by charge conjugation and composed of a quark d with an antistrange $\bar{s}$ and a $\bar{d}$ with a s respectively. Whilst the mass eigenstates are

$$|K_\mathrm{L}\rangle = \frac{p|K^0\rangle + q|\bar{K}^0\rangle}{\sqrt{|p|^2+|q|^2}} \quad (2)$$

and

$$|K_\mathrm{S}\rangle = \frac{p|K^0\rangle - q|\bar{K}^0\rangle}{\sqrt{|p|^2+|q|^2}}, \quad (3)$$

where $p = 1+\varepsilon$ and $p = 1-\varepsilon$ in terms of the (small) electroweek CP-violation parameter ε ($|\varepsilon| = (2.26 \pm 0.02)10^{-3}$). The K_L is the long living state, corresponding for $\varepsilon = 0$ to the $CP = -\mathrm{e}^{\mathrm{i}\alpha}$ eigenstate ($|K^0_-\rangle$) for which two pion decay is forbidden, and K_S is the short living state, corresponding for $\varepsilon = 0$ to the $CP = +\mathrm{e}^{\mathrm{i}\alpha}$ eigenstate ($|K^0_+\rangle$), for which two pion decay is allowed.

Claims that these experimental set-ups [20] could allow the elimination of the detection loophole in view of the high efficiency of particles detectors have also been made. However, we have shown [21] that due to the necessity of identifying the state through some of its decays (or interaction) and since the decay channel (and interaction) can depend on the value of the hidden variables, the detection loophole appears in this case as well.

The very recent experimental results of [23] (following some previous ones concerning kaons of [24]), with entangled pairs of $B^0\bar{B}^0$ mesons from $\Upsilon(4S)$ decay, giving a violation $S = 2.725 \pm 0.167_\mathrm{stat} \pm 0.092_\mathrm{syst}$ of the CHSH inequality $S < 2$, are therefore very interesting, representing a test of local realism for a new kind of particles, but they could not lead to an ultimate test.

In some other recent papers [15–18] the connection between the value of CP-violating parameters ε and ε' to some Bell inequality has been studied with the purpose of showing that the simple observation of some specific values of these parameters represent by itself a test of local realism.

Considering the large relevance of these results, in this paper we will carefully analyze the explicit and implicit hypotheses on which these inequalities are based showing that they do not represent a clear test of local realism, since other additional assumptions are needed. The results of [15–18] are therefore very interesting for having pointed out a connection between CP-violating parameters and local realism, but they do not allow a conclusive test of the latter one to be made.

2 Discussion of the connection between CP-violation parameters and local realism

Let us begin by considering the proposal of [17,18].

The main idea of these papers is considering a Clauser–Horne-like inequality (one of the many different forms of Bell inequalities [1]) on the joint probabilities of observing at a certain time t_1 the first K in a state f_1 ($-$ means no selection) and the second at t_2 in a state f_2:

$$\begin{aligned} &P(f_1,t_1;f_2,t_2) - P(f_1,t_1;f_4,t_2) \\ &\quad + P(f_3,t_1;f_2,t_2) + P(f_3,t_1;f_4,t_2) \\ &\qquad \le P(f_3,t_1;-,t_2) + P(-,t_1;f_2,t_2); \end{aligned} \quad (4)$$

afterwards the relation (4) is transformed in an inequality on the CP-violation parameter ε' (see for example [19] for its definition),

$$|\mathrm{Re}\{\varepsilon'\}| \le 3|\varepsilon'|^2. \quad (5)$$

In [18] is then shown how the data of the KTev [27] and NA48 [28] Collaborations (obtained with uncorrelated kaons) violate this inequality by some standard deviations.

However, a first caveat, already indicated in the paper, is that the demonstration is obtained with the assumption that the decay of one kaon is stochastically independent from the decay of the entangled one. In our opinion, this represents by itself a strong reduction of the generality of LHVT as tested by this scheme, since obviously in a deterministic theory also the decay channel can be fixed a priori by hidden variables.

[4] A recent experiment [14] based on the use of Be ions has reached very high efficiencies (around 98%), but in this case the two subsystems (the two ions) are not really separated systems during the measurement and the test cannot be considered a real implementation of a detection loophole free test of Bell inequalities, even if it could represent a progress in this sense.

However, we would like to emphasize that even larger loopholes remain in this kind of test of local realism.

First of all, it must be noticed that in a general LHVT framework the values extracted for ε' in uncorrelated kaons decay do not necessarily correspond to what one would obtain with correlated ones (this of course could eventually be tested in the future). Furthermore, by using the uncorrelated kaons results of [27,28], one does not cope at all with the problem of having space-like separated measurements in order to avoid a possible influence of the results.

Nonetheless, even more deep conceptual problems remain unsolved. In particular, we wish to point out that a further strong assumption appears, since, in transforming inequality (4) in (5), explicit values of SQM amplitudes are used (including also amplitudes for the unphysical (being CP-symmetry broken) $K^0_\pm$ states): these relations in principle could not be valid in a general LHVT. When building a suitable LHV theory one must satisfy the condition of reproducing SQM results in the sense of reproducing the good agreement of SQM predictions with *observed physical quantities* (within present uncertainties), but no condition is posed for unobserved processes.

In detail, in [17,18] the inequality (4) is rewritten for the probabilities $P(\pi^- l^+\nu, t; 2\pi^0, t)$, $P(\pi^- l^+\nu, t; 2\pi^+\pi^-, t)$ and $P(2\pi^0, t; \pi^+\pi^-, t)$ that are expressed by using quantum mechanics [25,26] in terms of the amplitudes r_{00} (the ratio of the decay amplitudes of K_+ and K_- into $\pi^0\pi^0$), r_{+-} (the ratio of the decay amplitudes of K_+ and K_- into $\pi^+\pi^-$), x (describing the violation of the $\Delta S = \Delta Q$ rule) and the parameters $\gamma_\mathrm{S}, \gamma_\mathrm{L}, \lambda_\mathrm{L}$ characterizing the eigenvalues of effective Hamiltonian describing the K^0 time evolution (see [17, 18] for definitions):

$$\begin{aligned} &P(\pi^- l^+\nu, t; 2\pi^0, t) \\ &= \frac{1}{4}\mathrm{e}^{-(\gamma_\mathrm{L}+\gamma_\mathrm{S})t}[1 - 2\mathrm{Re}(r_{00}) - 2\mathrm{Re}(x)], \\ &P(\pi^- l^+\nu, t; 2\pi^+\pi^-, t) \\ &= \frac{1}{4}\mathrm{e}^{-(\lambda_\mathrm{L}+\gamma_\mathrm{S})t}[1 - 2\mathrm{Re}(r_{+-}) - 2\mathrm{Re}(x)], \\ &P(2\pi^0, t; \pi^+\pi^-, t) = \frac{1}{2}\mathrm{e}^{-(\gamma_\mathrm{L}+\gamma_\mathrm{S})t}|r_{+-} - r_{00}|^2, \end{aligned} \tag{6}$$

and then in terms of ε' by using $r_{+-} = \varepsilon - \epsilon_\mathrm{L} + \varepsilon'$ and $r_{00} = \varepsilon - \epsilon_\mathrm{L} - 2\varepsilon'$, where ε and ε' signal CP- and CPT-violating effects and can be related to the ratios of the decay amplitudes of $K_\mathrm{L,S}$ in two pions through

$$\varepsilon + \varepsilon' = \frac{A(K_\mathrm{L} \to \pi^+\pi^-)}{A(K_\mathrm{S} \to \pi^+\pi^-)}$$

and

$$\varepsilon - 2\varepsilon' = \frac{A(K_\mathrm{L} \to \pi^0\pi^0)}{A(K_\mathrm{S} \to \pi^0\pi^0)}$$

(see again [17,18] for a definition of ϵ_L).

All these relations are calculated in SQM, but in a general LHVT they do not need to have the same form. When one wants to discuss in complete generality, like Bell inequalities, the possible existence of a local realistic theory, one must reproduce, within available uncertainties, the observed quantities as branching ratios, but all the other amplitudes or parameters and relations among them, specific for quantum mechanics, cannot be assumed. Thus, the results of [17,18], albeit representing an interesting connection between local realism and specific properties (CP-violation) of the electroweak lagrangian, do not lead to a resolutive test of local realism.

In this sense also the argumentation proposed in [18] as an indication of the difficulty of building a LHVT where the two entangled kaons decays are not stochastically independent is based on the use of quantum evolution and relations among quantum amplitudes and therefore, for the previous argument, is rather weak.

For the sake of completeness, let us also notice that a direct use of inequality (4) for joint probabilities measured with an entangled state (1) would unavoidably lead to the problem of a finite efficiency in detecting a certain state. Exactly as for the other Bell inequalities tests (with mesons [21], photons [11,13], etc.) this requires an additional hypothesis stating that the selected sample is a faithful representative of the whole if the total detection efficiency does not exceed the 82.84%, a condition difficult, if not impossible, to be realized experimentally.

Next we discuss the considerations expressed in [16]. This paper reconsiders the original proposal of [15] to build a Bell inequality based on joint measurement probabilities of a K^0_S, $\bar{K}^0$ and the (unphysical) K^0_+ with the state (1):

$$P(K^0_\mathrm{S}, \bar{K}^0) \le P(K^0_\mathrm{S}, \bar{K}^0_+) + P(K^0_+, \bar{K}^0), \tag{7}$$

and, with some additional hypothesis on the phases, to transform it into an inequality on the parameter ε:

$$\mathrm{Re}\{\varepsilon\} \le |\varepsilon|^2. \tag{8}$$

The inequality violated by the present data on ε [22], $\varepsilon = (2.284 \pm 0.014)10^{-3}\mathrm{e}^{\mathrm{i}(43^o.52\pm0.06)}$.

The main idea of [16] is to obtain an inequality independent by any phase convention. In order to derive this result (7) is rewritten by using the SQM probabilities

$$\begin{aligned} P(K^0_\mathrm{S}, \bar{K}^0) &= \frac{|p|^2}{2\sqrt{|p|^2 + |q|^2}}, \\ P(K^0_\mathrm{S}, \bar{K}^0_+) &= \frac{|p\mathrm{e}^{\mathrm{i}\alpha} - q|^2}{4\sqrt{|p|^2 + |q|^2}}, \\ P(K^0_+, \bar{K}^0) &= 1/4 \end{aligned} \tag{9}$$

in the form

$$|p| \le |q|. \tag{10}$$

The values of $|p|$ and $|q|$ extracted from semileptonic decays [22] violate this inequality.

Furthermore, by replacing $\bar{K}^0$ with K^0 in the Bell inequality (7) one arrives at the inequality

$$|p| \le |q| \tag{11}$$

that together with (10) implies $|p| = |q|$, in contradiction with experiment [22].

However, the result (10) is not free from additional assumptions beyond local realism.

Again the relations (9) between probabilities and parameters p, q are based on quantum mechanics (and two of them concern the unphysical state K^0_+). The same considerations about the criticality of this point for a LHVT discussed for the previous case remain valid.

Furthermore, the values of $|p|$ and $|q|$ are extracted by analyzing specific decays of $K^0_{\mathrm{L,S}}$. Again, if these values are not obtained by using space-like separated measurements on entangled kaons, the locality loophole is not coped with. Also, if they are extracted by specific decay channels, one can conceive LHVT where the hidden variables determines also the kind of decay and therefore the values of $|p|$ and $|q|$ determined in some specific decay are not general parameters pertaining an unbiased hidden variable sample, which should be used for the inequality (10).

Thus from all these considerations it follows that also the proposal of [16] is not free from loopholes.

3 Conclusions

In summary, the results of [15–18] represent an interesting connection between local realism and electroweak CP-violation, but they are not model independent. Further assumptions are needed for obtaining these results beyond local realism, and thus they cannot absolutely represent a conclusive test of this hypothesis, albeit giving interesting indications disfavoring LHVT. Furthermore, it must be noticed that, in this case, these additional assumptions are contained in the derivation of the relations (5), (8) and (10).

In particular we have shown how the detection loophole, that in this case should not simply be considered to refer to the efficiency in detecting a particle but also to the efficiency of tagging it, appears in a very severe way. This is an argument that has not been pointed out in previous papers.

Therefore, at variance with the case of a detection loophole for photon experiments [9] or a locality loophole for ions [14] that depend on technological limitations, it will not be possible to overcome this problem.

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References

1. M. Genovese, preprint IEN-12-04, Phys. Rep., DOI: 10.1016/j.physrep.2005.03.003
2. P. Ghose, Testing quantum mechanics on a new ground (Cambridge Univ. Press, Cambridge 1999)
3. G. Auletta, Foundations and interpretation of quantum mechanics (World Scientific, Singapore 2000)
4. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 77 (1935)
5. J.S. Bell, Physics **1**, 195 (1965)
6. E. Schrödinger, Proc. Camb. Phil. Soc. **31**, 555 (1935)
7. G. Brida et al., J. Phys. B At. Mol. Opt. Phys. **35**, 4751 (2002); Phys. Rev. A **68**, 033803 (2003)
8. A. Aspect et al., Phys. Rev. Lett. **49**, 1804 (1982)
9. J.P. Franson, Phys. Rev. Lett. **62**, 2205 (1989); J.G. Rarity, P.R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990); J. Brendel et al., Eur. Phys. Lett. **20**, 275 (1992); P.G. Kwiat et al., Phys. Rev. A **41**, 2910 (1990); T.E. Kiess et al., Phys. Rev. Lett. **71**, 3893 (1993); P.G. Kwiat et al., Phys. Rev. Lett. **75**, 4337 (1995); Z.J. Ou, L. Mandel, Phys. Rev. Lett. **61**, 50 (1988); Y.H. Shih et al., Phys. Rev. A **47**, 1288 (1993); G. Brida et al., Phys. Lett. A **268**, 12 (2000); **299**, 121 (2002); J.R. Torgerson et al., Phys. Lett. A **204**, 323 (1995); J.W. Pan et al., Nature **403**, 515 (2000)
10. W. Tittel et al., Phys. Rev. Lett. **81**, 3563 (1998); G. Weihs et al., Phys. Rev. Lett. **81**, 5039 (1998)
11. E. Santos, Phys. Lett. A **212**, 10 (1996); L. De Caro, A. Garuccio, Phys. Rev. A **54**, 174 (1996) and references therein
12. P.H. Eberhard, Phys. Rev. A **47**, R747 (1993)
13. A. Garuccio, Phys. Rev. A **52**, 2535 (1995)
14. M.A. Rowe et al., Nature **409**, 791 (2001)
15. F. Uchiyama, Phys. Lett. A **231**, 295 (1997)
16. R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, Phys. Lett. A **289**, 21 (2001); R.A. Bertlmann, quant-ph 0410028
17. F. Benatti, R. Floreani, Phys. Rev. D **57**, R1332 (1998)
18. F. Benatti, R. Floreani, Eur. Phys. J. C **13**, 267 (2000)
19. E. Leader, E. Predazzi, An introduction to gauge theoris and modern particle physics (Cambridge Univ. press, Cambridge, 1996)
20. P.H. Eberhard, Nucl. Phys. B **398**, 155 (1993); G.C. Ghirardi et al., The DAΦNE Physical handbook, edited by L. Maiani, G. Pancheri, N. Paver (INFN, Frascati, 1992), Vol. I; A. Di Domenico, Nucl. Phys. B **450**, 293 (1995); B. Ancochea, Phys. Rev. D **60**, 094008 (1999); A. Bramon, M. Nowakowski, Phys. Rev. Lett. **83**, 1 (1999); N. Gisen, A. Go, Am. J. Phys. **69**, 264 (2001); B.C. Hiesmayr, hep-ph 0010108; R.H. Dalitz, G. Garbarino, quant-ph 0011108; R.A. Bertlmann, W. Grimus, Phys. Lett. B **392**, 426 (1997); Phys. Rev. D **58**, 034014 (1998); G.V. Dass, K.V.L. Sarma, Eur. Phys. J. C **5**, 283 (1998); H. Albrecht, Phys. Lett. B **192**, 245 (1987); F. Selleri, Phys. Rev. A **56**, 3493 (1997); A. Pompili, F. Selleri, Eur. Phys. J. C **14**, 469 (2000); A. Bramon, G. Garbarino, Phys. Rev. Lett. **88**, 040403 (2002); Phys. Rev. Lett. **89**, 160401 (2002); P. Privitera, F. Selleri, Phys. Lett. B **296**, 261 (1992); R. Foadi, F. Selleri, Phys. Rev. A **61**, 012106 (2000)
21. M. Genovese et al., Phys. Lett. B **513**, 401 (01); Foud. Phys. **32**, 589 (2002); M. Genovese, Phys. Rev A **69**, 022103 (2004)
22. Particle Data Book, S. Eidelman et al., Phys. Lett. B **592**, 1 (2004)
23. A. Go, J. Mod. Opt. **51**, 991 (2004)
24. A. Apostolakis et al., Phys. Lett. B **422**, 339 (1998)
25. P. Huet, M.E. Peskin, Nucl. Phys. B **434**, 3 (1995)
26. F. Benatti, R. Floreani, Phys. Lett. B **401**, 337 (1997)
27. The KTev Collaboration, Phys. Rev. Lett. **88**, 22 (1999)
28. The Na48 Collaboration, Phys. Lett. B **465**, 335 (1999)